

$$(\ln x)\left(\frac{1}{x}\right)$$

$$\int \frac{\ln x}{x} dx =$$

$$u du$$

$$u = \ln x$$

$$= \frac{u^2}{2} + C$$

$$du = \left(\frac{1}{x} dx\right)$$

$$\cancel{x} du = dx$$

$$= \frac{(\ln x)^2}{2} + C$$

$$\frac{d}{dx} \left(\frac{2(\ln x)^2}{2} \right) = \frac{2(\ln x)'}{2} \left(\frac{d}{dx} \ln x \right) = \frac{\ln x}{x}$$

$$\frac{d}{dx} (f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$\int \underbrace{f'(g(x)) \cdot g'(x)}_{\text{red bracket}} dx = f(g(x)) + C$$

More complicated

Complicated anti-derivatives

$$\int \ln x \, dx$$

$$\begin{aligned} \frac{d}{dx} (x \ln x - x) &= (1)(\ln x) + x\left(\frac{1}{x}\right) - 1 \\ &= \ln x + 1 - 1 = \ln x \end{aligned}$$

Product Rule

$$\int \frac{d}{dx} (f \cdot g) = \int f' g + \int f g'$$

$$f \cdot g + C = \int f' g dx + \int f g' dx$$

$$+ C + f \cdot g - \int f' g dx = \int f g' dx$$

$$\int f g' dx = f g - \int f' g dx + C$$

$$\int \underbrace{u}_{\text{You}} \underbrace{dv}_{\text{Vee}} = \underbrace{uv}_{\text{you \cdot vee}} - \int \underbrace{v}_{\text{Vee}} \underbrace{du}_{\text{you}}$$

$$\int \ln x dx = x \ln x - \int (x) \left(\frac{1}{x} \right) dx$$

$$\begin{aligned} u &= \ln x & dv &= 1 dx \\ du &= \frac{1}{x} dx & v &= x \end{aligned}$$

$$\begin{aligned} &= x \ln x - \int 1 dx \\ &= x \ln x - x + C \end{aligned}$$

$$\int f g' dx = fg - \int f' g dx + c$$
$$\int u dv = uv - \int v du$$

$$\int x \ln x dx = \left(\frac{x^2}{2} \right) \ln x - \int \left(\frac{x^2}{2} \right) \left(\frac{1}{x} \right) dx$$
$$u = \ln x \quad dv = x dx$$
$$du = \frac{1}{x} dx \quad v = \frac{x^2}{2}$$
$$\frac{x^2}{2} \ln x - \int \frac{x}{2} dx$$
$$\frac{x^2}{2} \ln x - \frac{1}{2} \int x dx$$
$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\int x e^x dx =$$

$$u = x \quad dv = e^x dx$$

$$\underbrace{du = dx} \Leftrightarrow v = e^x$$

$$x e^x - \int e^x dx$$

$$= x e^x - e^x + C$$

$$\int x e^x dx =$$

$$u = e^x \quad dv = x dx$$

$$\underbrace{du = e^x dx} \quad \underbrace{v = \frac{x^2}{2}}$$

$$e^x \left(\frac{x^2}{2} \right) - \int \frac{x^2}{2} e^x dx$$

$$\int x \sin x \, dx$$

$$u = x \quad dv = \sin x \, dx$$
$$du = dx \quad v = -\cos x$$

$$= -x \cos x - \left(\int -\cos x \, dx \right)$$

$$= -x \cos x + \sin x + C$$

$$\int e^x \sin x \, dx =$$

$$u = \sin x \quad dv = e^x \, dx$$

$$du = \cos x \, dx \quad v = e^x$$

$$e^x \sin x - \int e^x \cos x \, dx = e^x \sin x - \left[e^x \cos x + \int e^x \sin x \, dx \right]$$

$$u = \cos x \quad dv = e^x \, dx$$

$$du = -\sin x \, dx \quad v = e^x$$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx + \int e^x \sin x \, dx$$

$$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

8.5/15) $\int \frac{x^2+2}{x+2} dx$

$$= \int x-2 + \frac{6}{x+2} dx$$

$$= \frac{x^2}{2} - 2x + 6 \ln|x+2| + C$$

$$\begin{array}{r} x-2 \\ x+2 \overline{) x^2+2} \\ \underline{-(x^2+2x)} \\ -2x+2 \\ \underline{-(-2x-4)} \\ 6 \end{array}$$

$$\int f(g)' = \int f'(g) \cdot g'$$

$$f(g) = \int f'(g) \cdot \cancel{g'(x)} dx$$

$$\int e^x (1+e^x)^5 dx = \int u^5 du$$

Let $u = 1+e^x$

$$du = e^x dx$$

$$= \frac{u^6}{6} + C$$

$$= \frac{(1+e^x)^6}{6} + C$$

$$\int x e^x dx$$

$$= x e^x - e^x + C$$

$$\frac{d}{dx}(x e^x - e^x + C) = (1)e^x + x(e^x) - e^x = x e^x$$

More complicated complicated
anti-derivatives

$$\int \frac{d}{dx} (f \cdot g) dx = \int f' g dx + \int f g' dx$$

$$fg = \int f' g dx + \int f g' dx$$

$$-\int f' g dx \quad | \quad -\int f' g dx$$

$$fg - \int f' g dx = \int f g' dx$$

$$\left(\begin{aligned} fg - \int f'g dx &= \int fg' dx \\ \int u dv &= uv - \int v du \end{aligned} \right)$$

$$\int (x e^x) dx = x e^x - \int e^x dx = \underline{x e^x - e^x + C}$$

$$u = x \quad dv = e^x dx$$

$$du = 1 dx \quad v = e^x$$

$$\int \ln x \, dx = x \ln x - \int x \left(\frac{1}{x} \right) dx$$

$u = \ln x$ $dv = 1 \, dx$

$du = \frac{1}{x} \, dx$ $v = x$

$$= x \ln x - \int 1 \, dx$$
$$= \underline{x \ln x - x + C}$$

Formula: $\int u \, dv = uv - \int v \, du$

$$\int u dv = uv - \int v du$$

$$\int x \ln x dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \left(\frac{1}{x} \right) dx$$

$u = \ln x \quad dv = x dx$
 $du = \frac{1}{x} dx \quad v = \frac{x^2}{2}$

$$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$$
$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\int x \cos x \, dx =$$

$$u = x \quad dv = \cos x$$

$$du = dx \quad v = \sin x$$

$$\begin{aligned} \int x \cos x \, dx &= x(\sin x) - \int \sin x \, dx \\ &= x \sin x + \cos x + C \end{aligned}$$

$$\int e^x \cos x dx$$

$$u = e^x \quad dv = \cos x dx$$

$$du = e^x dx \quad v = \sin x$$

$$\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx$$

$$u = e^x \quad dv = \sin x dx$$

$$du = e^x dx \quad v = -\cos x$$

$$\int e^x \cos x dx = e^x \sin x - \left[e^x \cos x + \int e^x \cos x dx \right]$$

$$\int e^x \cos x dx = e^x \sin x - e^x \cos x - \int e^x \cos x dx$$

$$+ \int e^x \cos x dx$$

$$\therefore 2 \int e^x \cos x dx = e^x \sin x - e^x \cos x$$

and

$$\int e^x \cos x dx = \frac{e^x \sin x - e^x \cos x}{2}$$

8.2/4

$$\int x^2 e^{-2x} dx$$

$$u = x^2 \quad dv = e^{-2x} dx$$

$$du = 2x dx \quad v = -\frac{1}{2} e^{-2x}$$

$$= -\frac{x^2}{2} e^{-2x} + \int \left(\frac{1}{2} e^{-2x}\right) (2x dx)$$

$$= -\frac{x^2}{2} e^{-2x} + \int x e^{-2x} dx$$

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$$= -\frac{x^2}{2} e^{-2x} + \left[-\frac{1}{2} x e^{-2x} - \int -\frac{1}{2} e^{-2x} dx \right]$$

$$= -\frac{x^2}{2} e^{-2x} - \frac{x}{2} e^{-2x} + \frac{1}{2} \left(-\frac{1}{2} e^{-2x} \right) + C$$

$$\int e^{-2x} dx$$

$$u = -2x$$

$$du = -2 dx$$

$$-\frac{1}{2} du = dx$$

$$-\frac{1}{2} \int e^u du$$

$$= -\frac{1}{2} e^u + C$$

$$= -\frac{1}{2} e^{-2x} + C$$

